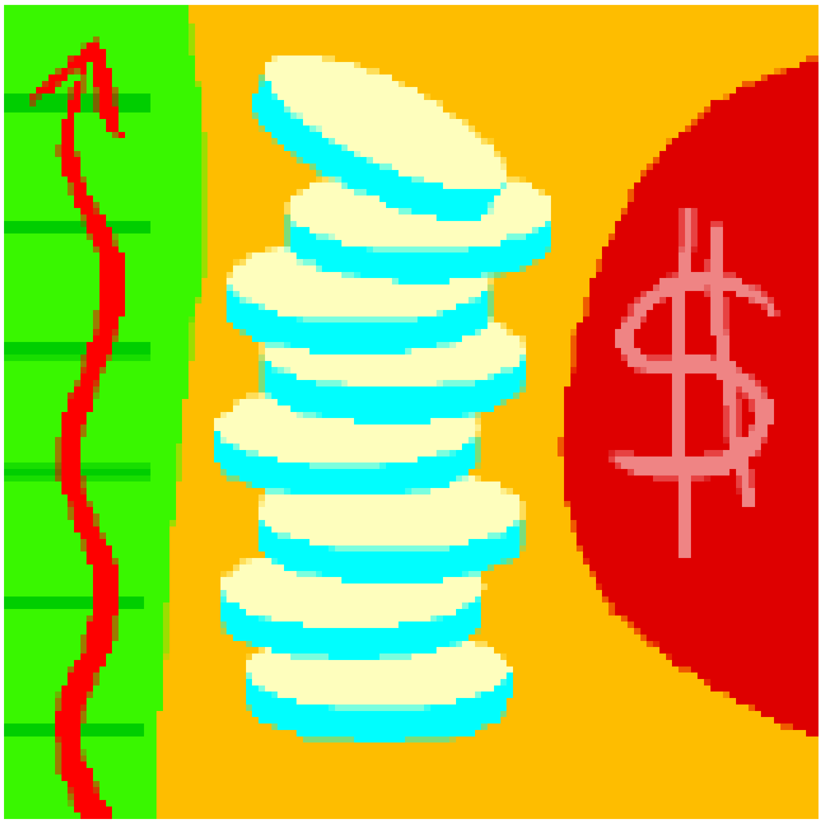
Quality Loss Function and Tolerance Design

A method to quantify savings from improved product and process designs



NOTICE

This is a shareware version of our seminar notes on the above topic. This document is placed in the public domain for free download & distribution by practitioners of quality improvement tools and techniques (QITT). You are allowed to make use of this documents as is and without any support.

Nutek, Inc. (Since 1987)

Quality Engineering Seminar and Software

Bloomfield Hills, MI, USA. 2014

Overview

Attempts to define quality have demystified quality experts of all times. While many have forwarded numerous interpretations of quality is all about, Dr. Genichi Taguchi of Japan has proposed an intriguing definition of quality that covers a broader aspects than others. Dr. Taguchi defined quality as the ill effects (loss to the society) of un-quality of products and services the entire life of service in the hands of consumers. He further offered a mathematical formulation to quantify the magnitude of such effects, which he calls loss to the society, in terms of monetary units. Today, for all activities engaged in quality improvement of products and processes, the proposed concept of *Loss Function* is used mainly in the following tow ways:

- Quantify and compare merits of design improvements in terms of monetary values.
- Fine tune designs (*Tolerance Design*) based on optimum conditions obtained by *Parameter Design*.

Introduction

Evaluating and establishing improvements we achieve in performance, cost or quality is common tasks we perform at end of a project completion. Generally, we also express such improvements achieved in terms of percentage of the current level of performance. However, when asked to quantify the same improvement in terms of dollars we seem to have difficult time. Fortunately, the Loss Function formulation proposed by Dr. Genechi Taguchi allows us to translate the expected performance improvement in terms of savings expressed in dollars. Using the Loss Function concept, the expected savings from the improvement in quality, i.e., reduced variation in performance around the target can be easily transformed into cost.

This session will present a quick review of the product and process design improvement using the Taguchi design of experiment (DOE) and discuss how the Loss Function is used to convert the improved performance in to expected dollar saving. Basic theories are covered and application demonstrated through examples.

You would benefit most from this sessions if you have working knowledge of the DOE/Taguchi approach and wish to express improvements in terms of dollars.

TOPICS OF DISCUSSIONS

1. INTRODUCTION

    - Concepts
    - Math model
    - Purpose and application areas

2. EVALUATION OF $ LOSS

    - With target value
    - General quality characteristics
    - When other distribution parameters are known

3. COMPUTATION OF SAVINGS FROM CHANGES IN

    - S/N ratio
    - Mean Squared Deviation (MSD)
    - Std. Deviation and Average Value

4. RELATIONSHIPS BETWEEN LOSS AND PROCESS CAPABILITY (cpk)

5. GENERAL APPLICATION EXAMPLES

6. APPLICATION TO ACTUAL PROJECTS

SYMBOLS AND ABBREVIATIONS

Y = Measured value of quality characteristic (QC)

Y_a = Average value of measured quality characteristic

Y_o = Target value of quality characteristic

Tol = Tolerance of Y (in case of Nominal the best)
T_c = Consumer Tolerance (in case of Bigger and Smaller characteristics)

Y_{min} = Minimum value of Y

Y_{max} = Maximum value of Y

UCL = Upper control limit

LCL = Lower control limit

SD_c = Standard déviation (conventional definition)

SD_T = Standard deviation (used in Taguchi method)

MSD = Mean squared deviation

S/N = Signal to ratio of measured data (QC)

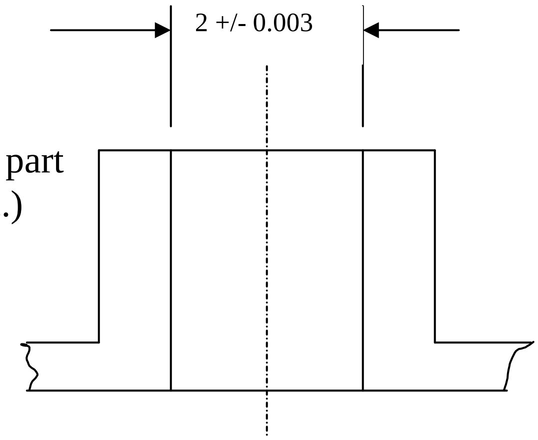
C_{pk} = Performance index

L = Loss in terms of dollars
L_c = Consumer Loss (cost of rejects when part is out of tolerance)

K = A constant dependent on production parameters

SQRT = Square root of the quantity within {}

1.0 INTRODUCTION



Loss = $ value of a nonfunctioning single part
(cost of rejects, rework, warranty, etc.)

How to calculate loss?

Conventional practice:

Consider a BEARING HOUSING (Part)

FIG. 1 Bearing Housing

Process

Machine bearing seat to 2 +/- .003”

Cost of machining each piece = $ 18.00/part

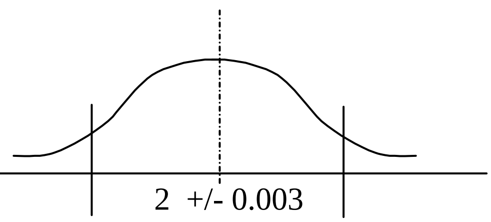


FIG. 2 Tolerance Limits

Part dimension (Y)	Loss (L)
2.0001	0
2.0015	0
1.998	0
2.004	$18.00
1.999	0
1.996	$18.00

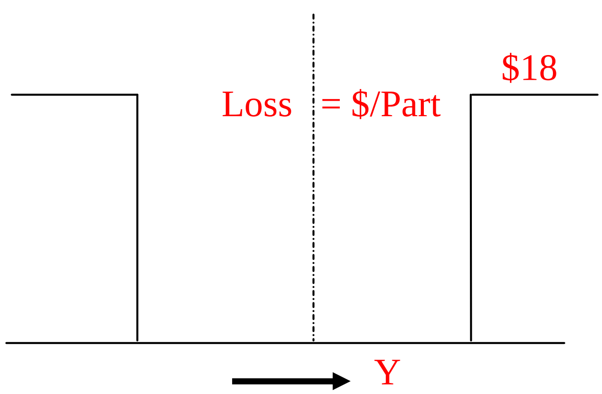


FIG. 3 Loss with Specification Limits

KEY OBSERVATIONS

Conventional practice:

1. If all parts are made within specification limits (SL), say at 2.001, theoretically, no part is wasted and thus no loss.
2. If all parts are made at or outside the SL, the entire cost of making a part is attributed to loss.

Note: From no loss at just before 2.003 to $18 loss at just over 2.003 produces the step jump in loss (see FIG. 3)

Taguchi Approach (Loss Function)

There is loss even if the part is made within the SL. In other words there is loss as long as the part deviates from the target.
(There is a warranty cost even if all parts are made to specification. Why?)

Now let's compare the two schools of thought.

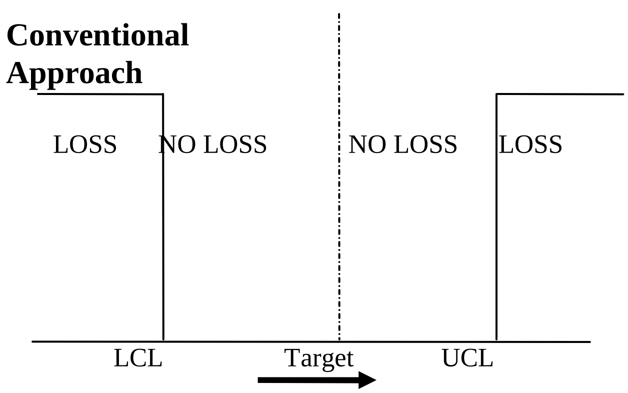


FIG.4 Loss Zones with Specification Limits

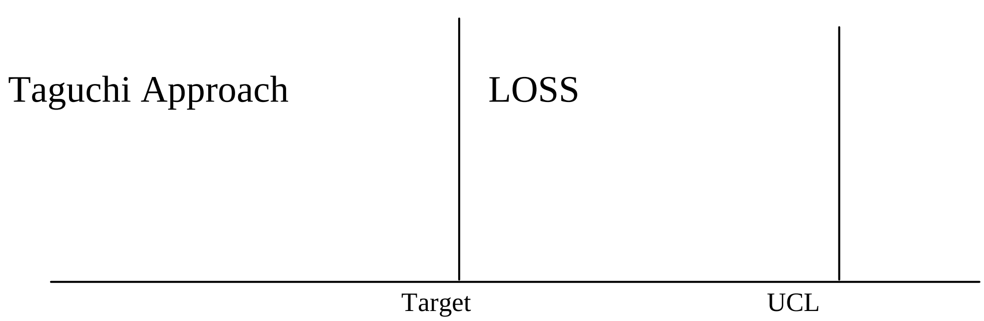


FIG. 5 Loss zone with Taguchi Approach

Note: Loss at $Y = Y_o$ =/- Tol (At UCL) is same for both.

Basis for mathematical formulation -

Assumption: Loss is continuous from target to UCL or LCL.

It can be linear, quadratic or higher order.

Taguchi formulated something that's not relatively simpler yet worked well. His choice to express loss as

$L = K\,(Y - Y_o)^2$ where K is a constant

Consider the case of Bearing housing machining process

$Y_o = 2.000$, Tol = .003

Cost of rejection, i.e., Loss when Y = UCL or LCL = $18.00

Since UCL = Y_o + Tol

Therefore $L = K\,(Y_o + \text{Tol} - Y_o)^2$ at point A

or $18 = K(.003)^2$

or $K = 2.00 \times 10^6$ $/sq. in

or $L = 2.00 \times 10^6\,(Y - Y_o)^2$

Thus when Y = 2.002 L = $2.00 and for Y = 2.003 L = $ 18.00

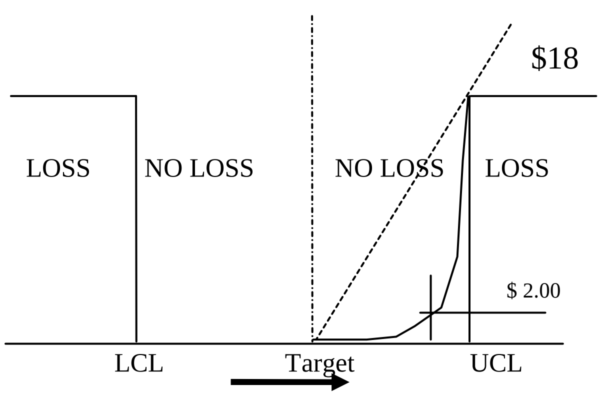


FIG . 6 Loss Function

Examine how the loss formula works.

$L = K\,(Y - Y_0)^2$

Assume
Y = 5, tol = .02

Cost of reject = $6.00

Thus

$$6 = K\,(Y_o +/- .02 - Y_o)^2 \quad \text{or} \quad K = 6/(.02 \times .02) = 15{,}000$$

or $\quad L = 15{,}000\,(Y - Y_0)^2$

Thus when Y = 5.005 L = $15{,}000\,(5.005 - 5.00)^2 = 0.375$

etc.

Y	$ Loss
5.00	0
5.005	0.375
5.01	1.500
5.015	3.375
5.02	6.000

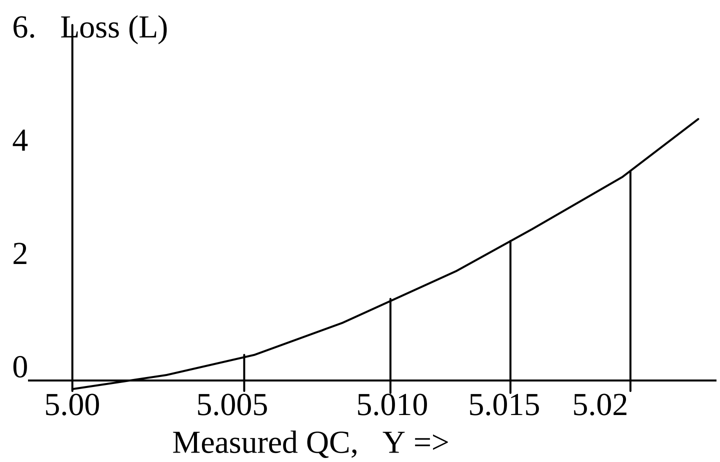


FIG. 7 Loss with Positive Deviations

Terms and Units of Loss Parameters

L + Loss is always expressed in terms of cost per unit of part
Loss is same as cost of rejection when a part is made at UCL
$0 < L <$ cost of rejection/making one part

Y = Measured value of part QC, any defined units

K = A constant. Unit is in $/Sq.in

Tol = UCL or LCL - Y_o

Y_o = Target value of QC (in the same unit as Y)

Total or average loss for a population of part:

For a population of part produced at different dimensions, say, Y_1, Y_2, Y_3, etc. average loss per product can be express as

$$L = K\left[\left(Y_1 - Y_o\right)^2 + \left(Y_2 - Y_o\right)^2 + \cdots\cdots + \left(Y_n - Y_0\right)^2\right] / N$$

or

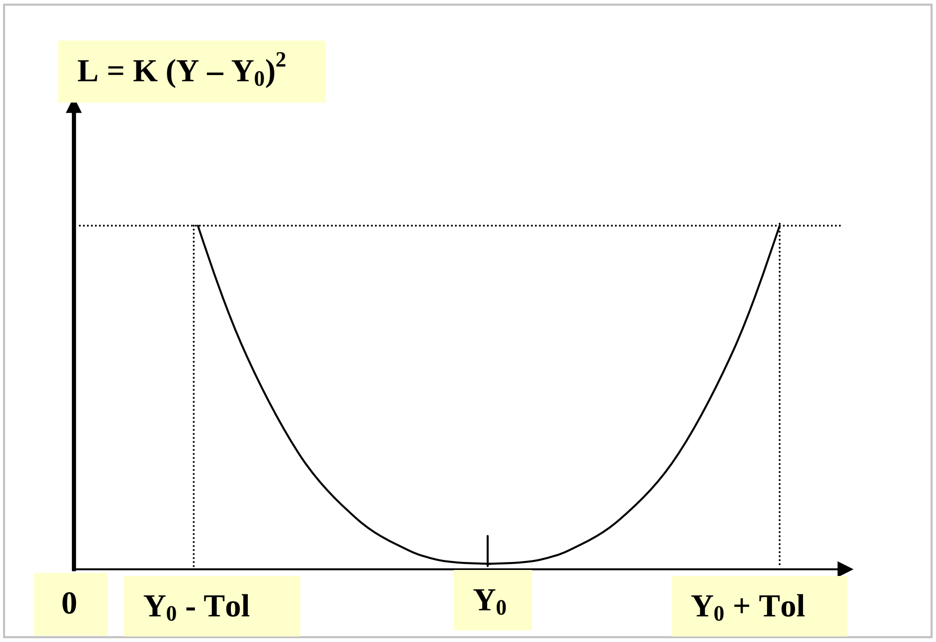
$$L = K\ (MSD) \quad \text{[by definition of MSD]}$$

Note:
1. L is always expressed as amount per part regardless of single or multiple unit of part information. If there is more than one part involved then instead of $(Y - Y_o)^2$ MSD of the parts is used.

2. K = L/MSD or $L/(Tol)^2$ implies that K should be found from either expression depending on the information available.

3. If tolerance is known then L = cost of rejection. When MSD is known loss needs to be computed from total cost of rejection and production rate.

Loss Function – *Nominal the best*



$$L = \begin{cases} K(Y - Y_0)^2 & \text{for one piece of data} \\ K(\text{MSD}) & \text{for multiple (n) pieces of data} \end{cases}$$

Where $K = L_c / Tol^2$

$$\text{MSD} = \sum_{i=1}^{n} (Y_i - Y_0)^2 = \frac{1}{n}\left[(Y_1 - Y_0)^2 + (Y_1 - Y_0)^2 \;\ldots\; + (Y_n - Y_0)^2\right]$$

This can be shown to be,

$$\text{MSD} = \sigma^2 + (Y_a - Y_0)^2$$

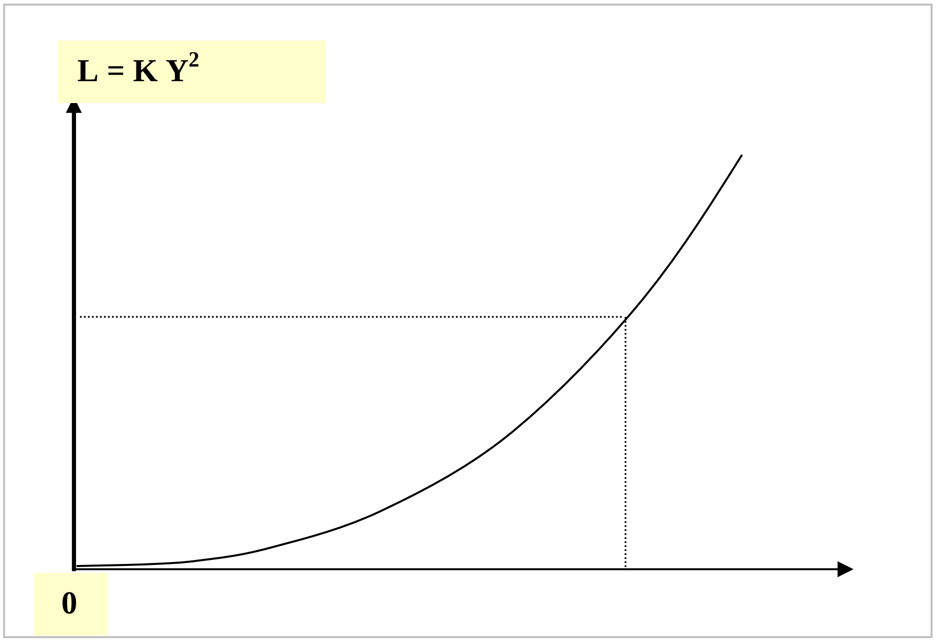
Or

$$L = K[\sigma^2 + (Y_a - Y_0)^2]$$

Variability around average

Variability of the average from the target

Loss Function – *Smaller is better*



$$L = \begin{cases} K\,(Y)^2 & \text{for one piece of data} \\ K\,(MSD) & \text{for multiple (n) pieces of data} \end{cases}$$

Where $K = L_c / T_c^{\,2}$

$$MSD = \sum_{i=1}^{n} (Y_i)^2 \qquad \frac{1}{n}\left[Y_1^{\,2} \quad + \quad Y_1^{\,2} \quad \ldots \quad + \quad Y_n^{\,2} \right]$$

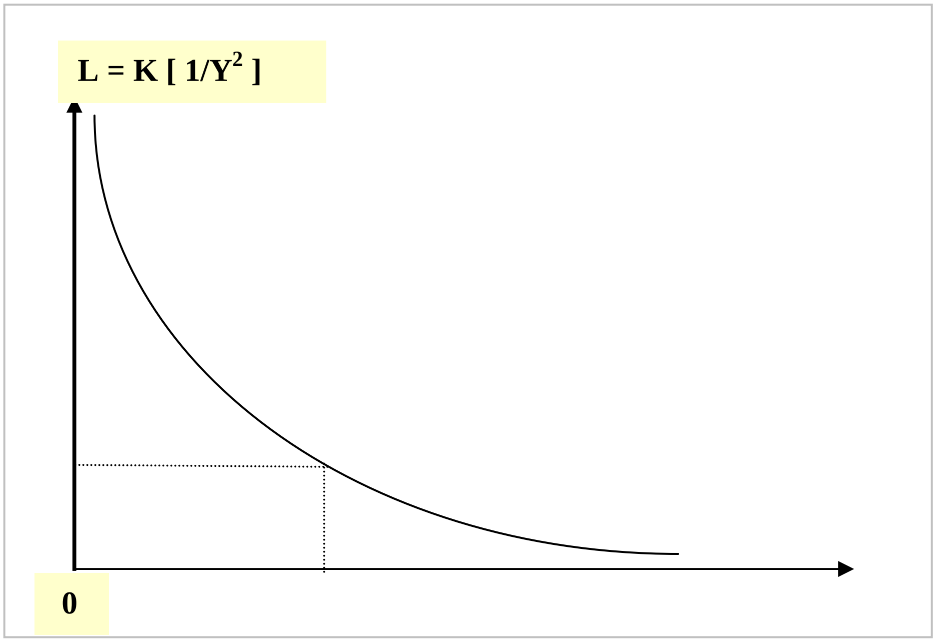
This can be shown to be,

$MSD = \sigma^2 + Y_a^{\,2}$

Or

$$\mathbf{L = K\,[\sigma^2 + Y_a^{\,2}]}$$

Loss Function – *Bigger is better*



$$L = \begin{cases} K\,(1/Y)^2 & \text{for one piece of data} \\ K\,(\text{MSD}) & \text{for multiple (n) pieces of data} \end{cases}$$

Where $K = L_c\ T_c^{\,2}$

$$\text{MSD} = \sum_{i=1}^{n} \frac{1}{Y_i^2} = \frac{1}{n}\left[\frac{1}{Y_1^2} + \frac{1}{Y_2^2} + \frac{1}{Y_3^2} \ \cdots \ + \frac{1}{Y_n^2}\right]$$

2.0 EVALUATION OF LOSS

EXAMPLE 1

A machine makes 5000 parts at a cost of $3.00 each.
Upon inspection 200 parts are rejected. Determine the loss per part.

SOLUTION

Total cost of rejection = 200 x $3.00 = $600

Average loss per part (L) = 600/5000 = $0.12/part

(If all 500 parts were rejected, then loss (L) would become
L = 3 x 5000/5000 = $ 3.00/part)

EXAMPLE 2

A production process makes batteries for 9 +/- .25 volts applications at a cost of $0.75 each. Determine:

a. Complete expression for loss function
b. Loss when a part is made at 9.10

SOLUTION

a) $L = K (Y - Y_o)^2$

when $Y = Y_0$ +/- .25 (at UCL or LCL) L = $.075

Thus $.75 = K(.25)^2$ or $K = .75/.0625 = 12$

or $L = 12 (Y - Y_0)^2$

b) $L = 12 (9.10 - 9.0)^2$ = $.12/part

EXAMPLE 3

A machine produces 10,000 6” long coil springs per day at a current production loss of \$.30/part (overhead & rejection). Samples examined before and after improvement are as follows:

Length BEFORE: 6.1 5.8 6.3 6.4 5.7 (5 examples)

Length AFTER: 6.15 6.2 5.9 6.1 6.05 5.8 (6 examples)

Calculate expected savings.

SOLUTION

BEFORE: MSD = $[(6.2 - 6.0)^2 + (5.8 - 6.0)^2 + ...]/5 = .0772$

$S/N = - 10 \log (.0772) = 11.12$

$L = K (MSD)$

or

$K = L/MSD = .30/.0772 = 3.886$

or

$L = 3.886 (MSD)$

AFTER: MSD = $[(6.15 - 6.0)^2 + (5.8 - 6.0)^2 + ...]/5 + .0772$

$S/N = 16.76$

Since L = 3.886 (MSD) (constant K found earlier)

L = 3.886 x .021 = \$.08/part

Therefore, Savings = (loss before - loss after)x monthly production

= (.30 - .08) x 10,000

= \$2,200/per month

EXAMPLE 4

A newly installed machine has the following performance characteristics:

Part specifications: Y = 12 +/- .70, Cost per part = $1.50

production rate = 60,000 units/month

Last month's production: Std. Dev. = .25, Y_a = 11.90 (10 samples)

Current production: Std. Dev. = .21, Y_a = 11.95 (12 samples)

Determine saving expected.

SOLUTION

$L = K (Y - Y_o)^2$ or $1.5 = K(O.70)^2$

or $K = 1.5/(.70 \times .70) = 3.06$

Thus

$L = 3.06 \times (Y - Y_o)^2$ also $L = K$ (MSD)

Last month: $MSD = SD_T^2 + M^2$

$= SD_c^2 \times (N - 1)/N + M^2$

$= (.25 \times .25)(10 - 1)/10 + (12 - 11.9)^2$

$= .066$

Thus, loss $L = 3.061 \times (.066) = \$0.20$

Current condition: $MSD = (.21 \times .21) \times (12 - 1)/12 + (.05 \times .05)$

$= .043$

Thus, loss $L = 3.061 \times .043 = \$0.13$

Therefore, savings = (0.20 - 0.13) x 60,000 = $4,200/month

3.0 COMPUTATION OF SAVING FROM IMPROVEMENT IN S/N

EXAMPLE 5

Old production status: Loss = $.20/part, MSD = .75
New production status: MSD = .50
Find $ savings as a percentage of loss in old production method.

SOLUTION

L = K (MSD)

Thus K = L/MSD = .20/.75 = 0.266 (from old production data)

or L = 0.266 x (MSD)

Loss in new production L = .266 x .50 = $ 0.133

% Savings = 100 x (Old loss - new loss)/old loss

= 100 x (.20 - .133)/.20

= 33.5%

EXAMPLE 6

A newly purchased milling machine was credited with improvement in S/N ratios of the machined parts from -18.5(before) to -12.5 (after).

Determine the expected savings as a percentage of the loss before improvement.

SOLUTION

let SN2 = S/N (before) = -18.5 and SN1 = S/N (after) = -12.5

% Savings = $[1 - 10^{-(SN2 - SN1)/10}] \times 100$

$= [1 - 10^{-(12.5 - (-18.5))/10}] \times 100$

$= [1 - 10^{-0.6}] \times 100$

$= [1 - .251] \times 100$

= 74.88%

4.0 RELATIONSHIPS BETWEEN LOSS AND PROCESS CAPABILITY INDEX

CONTROL CHARACTERISTIC (C_{pk})

$$C_{pk} = \frac{(Y_a - Y_{min})}{3 \text{ x } SD_c} \qquad \text{if } Y_a < Y_o$$

$$C_{pk} = \frac{(Y_{max} - Y_a)}{3 \text{ x } SD_c} \qquad \text{if } Y_a > Y_o$$

When Stand deviation is not known, but other production parameters are known, then C_{pk} can be calculated using the following relations

$M = Y_o - Y_a$

$SD_t =$ SQRT [(N -1)/N} x Sd_c

where SQRT = Square root {}

$MSD = Sd_t^2 + M^2$

S/N = - 10 log (MSD)

Loss L = K (MSD)

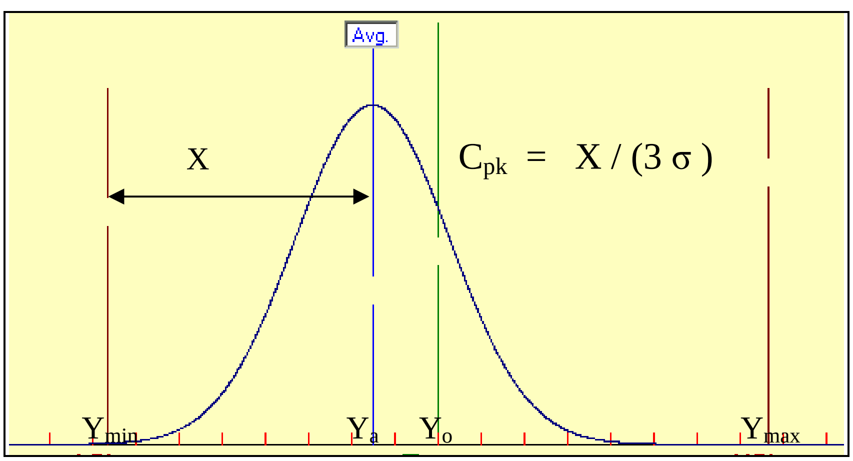


FIG. 8 C_{pk} FORMULATION

TABLE 1. RELATIONSHIPS AMONG COMMON PRODUCTION PARAMETERS

Assume that Y_o = 40, Y_{max} = 45, Y_{min} = 35, N = 10
Cost of rejection = $2.50
and Loss constant K = 2.50/(5x5) = 0.10

For a given set of Y_a and Sd_c, all other parameters shown in the table can be calculated as shown below.

Y_a(1)	SD_c(2)	C_{pk}(3)	SD_t(4)	M(5)	MSD(6)	S/N(7)	$L(8)
39	1.0	1.33	0.949	1.00	1.9006	-2.79	0.190
39	1.5	0.89	1.423	1.00	3.027	-4.81	0.300
39.5	1.0	1.50	0.949	0.50	1.151	-0.61	0.110
40.25	0.5	3.17	0.474	0.25	0.288	5.41	0.029
40	0.5	3.33	0.474	0.000	0.225	6.48	0.022
40	0.25	6.67	0.237	0.00	0.056	12.50	0.005

Sample calculation (first row of table 1)

Given Y_a = 39 and Sd_c = 10

(3) C_{pk} = (39 -35)/(3x1) = 4/3 = 1.33

(4) $SD_T = SD_c SQRT\ \{9/10\} = 0.949$

(5) M + $Y_o - Y_a$ = 40 - 39 = 1.0

(6) MSD = $SD_T^2 + M^2$ = (.949 x .949) + (1 x 1) = 1.900

(7) S/N = - 10 log (MSD) = -2.79

Using the value of loss constant, K, calculated above
(8) L = K (MSD) = 0.10 x 1.900 (K = .10)

APPENDIX A

DEVELOPMENT OF RELATIONSHIPS AMONG VARIOUS LOSS STATISTICS

By definition

Standard deviation $SD_c =$ ....(1)

Mean Squared Deviation MSD = ... (2)

In Taguchi approach, variation around the target is calculated as (Total sum of squares)

$S_T = \quad (Y_i - Y_o)^2$

$=$

$=$

$=$

$= N \times SD_T^2 + N \times M^2$

$S_T = N \times (SD_T^2 + M^2)$ .... (3)

where $M = (Y_a - Y_o)$ $N \times SD_T^2 = (Y_i - Y_a)^2$

that is $SD_T =$ .... (4)

(Standard deviation as used in Taguchi approach which differs from the standard definition by a factor square root of. (N-1/N)

Comparing the definitions of S_T and MSD,

$S_T = N \times MSD$ .... (5)

therefore, $MSD = SD_T^2 + M^2$ .... (6)

5.0 GENERAL APPLICATION EXAMPLES

(Workout examples to facilitate application of certain formulae)

6.0 APPLICATION CASE STUDIES

(Problems specific to the organization. Projects/data attendees bring to the class)

VII. EXERCISES (ON LOSS FUNCTION AND C_{pk})

For a machining operation the conditions presented as "GIVEN" are known. Calculate the quantities listed under "FIND". The answers are as shown within ( ). Assume N = 24 if unspecified.

1. a) GIVEN: $SD_c = 2$, $N = 20$ FIND: $SD_T = ?$ (1.924)

   b) GIVEN: $SD_c = 0.4$, $N = 12$ FIND: $SD_T = ?$ (3.830)

2. a) GIVEN: $SD_T = 0.32$, $N = 8$ FIND: $SD_c = ?$ (0.342)

   b) GIVEN: $SD_T = 6.0$, $N = 35$ FIND: $Sd_c = ?$ (6.088)

3. a) GIVEN: MSD = 0.0025 FIND: S/N = ? (26.020)

   b) GIVEN: MSD = 135.5 FIND: S/N = ? (-21.32)

   c) GIVEN: MSD = 2.5 FIND: S/N = ? (-3.980)

4. a) GIVEN: S/N = -15.0 FIND: MSD = ? (32.6228)

   b) GIVEN: S/N = 0.50 FIND: MSD = ? (0.8913)

   c) GIVEN: S/N = 3.20 FIND: MSD = ? (0.4786)

   d) GIVEN: S/N = 12.5 FIND: MSD = ? (0.0562)

5. a) GIVEN: $SD_T = 2.5$, FIND: MSD = ? (6.8125)
   M = 0.75 S/N = ? (-8.33)

   b) GIVEN: $SD_T = 0.03$, FIND: MSD = ? (0.0011)
   M = 0.015 S/N = ? (29.49)

6. a) GIVEN: $SD_c = 95$, FIND: MSD = ? (8561.9)
   M = 17 N = 12 S/N = ? (-39.33)

   b) GIVEN: $SD_c = 0.03$, FIND: MSD = ? (0.0013)
   M = .02 N = 24 S/N = ? (28.29)

7. a) GIVEN: $SD_c = 0.5$ LCL = 8 FIND: $C_{pk} = ?$ (0.57)
   $Y_o = 9.00$ $Y_a = 8.85$ S/N = ? (6.06)

EXERCISE (Cont'd.)

b) GIVEN: $SD_c = .03$ UCL = 6.1 FIND: $C_{pk} = ?$ (0.67)
$Y_o = 9.00$ $Y_a = 8.85$ S/N = ? (6.06)

c) GIVEN: $SD_c = 4.25$ LCL = 6.1 FIND: $C_{pk} = ?$ (1.65)
$Y_o = 225$ $Y_a = 221$ S/N = ? (-15.09)

d) GIVEN: $SD_c = 3.5$ LCL = 200 FIND: $C_{pk} = ?$ (2.00)
$Y_o = 225$ $Y_a = 221$ S/N = ? (-14.32)

e) GIVEN: $SD_c = 3.5$ LCL = 200 FIND: $C_{pk} = ?$ (2.29)
$Y_o = 250$ $Y_a = 226$ S/N = ? (-10.80)

8. a) GIVEN: S/N = 8.75 UCL = 9.50 FIND: $C_{pk} = ?$ (0.30)
$Y_o = 9.0$ $Y_a = 163$ MSD = ?(0.1334)

b) GIVEN: S/N = -12.5 UCL = 175 FIND: $C_{pk} = ?$ (1.28)
$Y_o = 160$ $Y_a = 163$ MSD = ?(17.783)

9. a) GIVEN: MSD = 3.2 UCL = 85 FIND: $C_{pk} = ?$ (0.83)
$Y_o = 80$ $Y_a = 80.75$ S/N = ? (-5.05)

b) GIVEN: MSD = 1.3 UCL = 85 FIND: $C_{pk} = ?$ (1.56)
$Y_o = 80$ $Y_a = 80.75$ S/N = ? (-1.14)

c) GIVEN: MSD = 3.2 UCL = 90 FIND: $C_{pk} = ?$ (1.80)
$Y_o = 80$ $Y_a = 80.75$ S/N = ? (-5.05)

10. a) GIVEN: S/N (before) = -13.2, S/N (after) = -8.75
FIND: Savings as a % of loss before experiment. (64.11%)

b) GIVEN: S/N (before) = 5.5 S/N (after) = 8.9
FIND: Savings as a % of loss before experiment (54.29%)

c) GIVEN: S/N (before) = -2.1 S/N (after) = 3.7
FIND: Savings as a % of loss before experiment (73.70%)

PRACTICE PROBLEMS (Loss Function Seminar)

1.1
A milling machine (M1) produces specimens whose surface finish readings (Bigger is better) are as follows:

8, 7, 8.5, 9.5, 9.3, 7.5, 8.6 (7 samples)

Find:
a) Average value [8.34]
b) Standard Deviation [classical, .910]
c) Standard Deviation - Taguchi form [.710]
d) MSD [.0148] d) S/N [18.28]

1.2
The machine (M1) in problem 1.1 produces 1,500 parts per month at a cost of $175 per part. Historically 30 parts/month are returned by the assembly division which makes use of the part.

Determine:
a) Loss per part [$3.5]
b) Monthly loss [$5250]
c) Loss constant, K [Ans: 3.5 / 0.148 = 236.4, MSD = 0.0148]

1.3
To improve the milling process, the machine of problem 1.1 was equipped with a new tool holder at a cost of $3,500. The new process produced the following samples:

8.8, 9.5, 8.6, 8.9, 7.9, 9.2, 9.4 and 8.6

Determine:
a) MSD [.01286]
b) Loss at improved condition [L = K MSD =……. = $3.04/part]
c) Monthly Loss [$4560]
d) Monthly savings [$630]
e) Effective annual savings [$630x12 - 3500 = $4060]

2.1
A manufacturer of a small servo motor spends $4,500 to reduce the operating noise of his product. The following noise reading (dB) were available from monthly production volume of 6,000 units. The total warranty and reject cost for the month $2500.

Old Process: 75, 72, 67, 81, 80 and 74
New Process: 76, 66, 63, 58, 67, 56 and 60

Find:

a) Loss before improvement [$.416/unit]
b) MSD before [5622]
c) Loss constant, K [7.4 x 10^-5]
d) MSD after [4098]
e) Loss after..[$.304/unit]
f) Savings [$673]
g) Return on investment (ROI) [$6.68 months]

2.2

Another sample taken from the improved process of problem 2.1 shows a S/N = -35.00. Using the same Loss constant, K = 7.4x10^-5, determine:

a) Loss associated with this sample ($.234/part)
b) If the cost to produce one motor is $25, what is the expected number of rejects out of monthly production of 6,000 units [56].

3.1

A machine shop supplies castings for inner race of a certain roller bearing of size 12.50 +/- .200. The shop produces 15,000 units monthly at a total cost $45000. A sample part dimensions were as follows:

12.5, 12.4, 12.6, 12.55 and 12.7

Find:

a) S/N	*[19.45]*
b) Loss constant	*[3/(0.2)^2 = 75]*
c) Loss	*[L = K MSD = 75 x 106-1.9 =$.851/unit]*
d) Loss per month	*[0.851 x 15000 = $12,769]*
e) Percent of part out of tolerance	*[28.36]*

3.2

To improve the casting process of problem 3.1, the owner purchased a new molding equipment at a cost of $76,000. The new mold produced casting which shows S/N = 26.5.

Determine:

a) Loss [$.167]
b) Monthly savings ($10,260)
c) ROI [7.4 months]
e) % of parts out of specs [5.56%]

3.3
A manufacturer of Tennis rackets intends to set 45 Lbs. tension in one of his product lines. The company spends $80,000 for total warranty (merchandise returned for out of specs tension). A sample data from monthly production of 35000 rackets show the following readings:

43, 48, 52, 44, 45, and 46 Lbs.

To improve the product, the owner invested $250,000 in a new assembly operation which produced the following samples:

46, 43, 44, 45, 44, 44.5 and 45 Lbs.

Determine:

a) Loss constant *[(80000/35000)/10.66) = .214]*
b) Loss from new production *[0.214 x 1.03 = $.22/unit]*
c) New savings during the year.

[(8000-7700) x 12 - 25000 = 614,500]

4.1
Cannons from three test tanks were shooting for a target 32,000 feet away. Determine the statistically superior performer by comparing their S/N (consider a sample of 10 shots for each).

	Cannon A:	Cannon B:	Cannon C:
Average	32,200 feet	32,100	32,150
Std. Dev.	110 feet	200	120

FIND S/N Cannon A: Cannon B: Cannon C:

Ans: [cannon C:S/n = -45, -45 > -46.89 > -47.17]

4.2
A pumpkin farmer finds his yield bringing the most returns when all pumpkins are of 20 Lbs. In the year 1989 his crop averaged 18 Lbs. with a Std. Deviation of 2.5 Lbs.
His loss in that year due to unsold pumpkins was $2,400. The following year, the farmer employed a new agricultural method which improved yield to average 19 Lbs with Std. Deviation of 2.0. Determine the savings expected to result as a percent of loss in 1989.
[Ans: 52.2%, 2,400 x 52.2 = $1,252.80]

5.1 A 25 cm gun barrel which bears a machining specification 25 +/- .250 costs $450 per piece to fabricate. Based on a sample of 20, the following statistics were computed:

Average = 25.080 and Std. Dev. = .045

Determine:

a) Cpk [1.26]
b) Loss function [L = 7200 $(y-25)^2$]
c) Std. Dev.-Taguchi [.044]
d) MSD [.044]
e) S/N [20.788]
f) Loss [$60/unit]

Answers:

(a) Cpk = (25.25 -25.080) /(3x o.o45) = 1.26
(b) K ==450 / (.250)^2 = 7200
(c) 0.045 (19/20)^.5 = 0.044
(d) (.044)^2 + (25.080 – 25)^2 = 1.936 x 10^-3 + 6.4 x 10^-3 = 0.00834
(e) S/N = - 10 Log (0.00834) = 20.788
(f) L = K MSD = 7200 x 0.00834 = $ 60/unit

TOLERANCE DESIGN

by

Ranjit K. Roy, Ph.D.,P.E., PMP
Nutek, Inc.
Bloomfield Hills, Michigan, USA.
E-mail: Support@Nutek-us.com Web Site: http://Nutek-us.com

Overview

Tolerance Design is the third and final phases in the Taguchi quality improvement strategies. Starting with the System Design at the concept level that follows the Parameter Design, Tolerance Design is used to put the final touch by fin tuning the optimum factor selections identified in the previous phase.

In *Tolerance Design*, the system is studied using current tolerances (factors at 2 or 3 levels) to determine which one to tighten and which one to be left alone. This is done by comparing the savings from quality improvement with the cost of upgrade.

Approach

Tolerance Design makes use of experimental designs with orthogonal arrays and the Taguchi Loss Function. The following are common steps in *Tolerance Design*.

Step 1: Run experiments setting factors at current tolerance limits and perform ANOVA.

Step 2: Calculate loss for the current system performance [L = K (MSD), or $L = K\ V_T$, where $V_T =$ ST /(DOF)$_T$]

Step 3: Determine potential savings (loss after – loss before) by adjusting factor levels:

- *Calculate current loss using ANOVA (P%)*
- *Estimate the new loss with upgraded factor level (better grade of materials, part, etc.)*

Step 4: Calculate net gain (4) from savings from all factors at the improved condition.

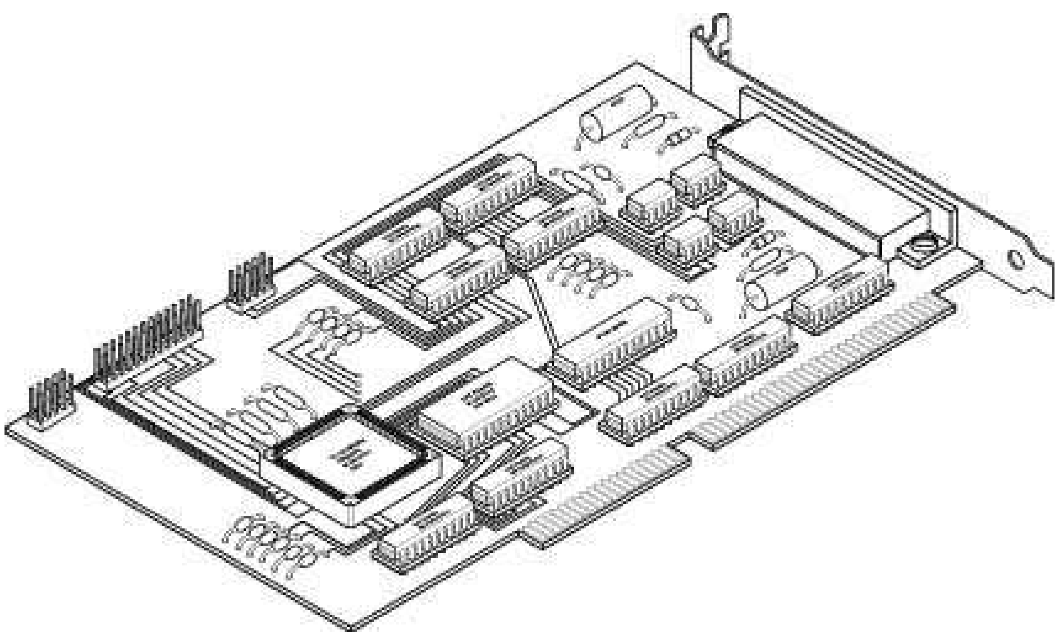
Example:
In a design for display control by a **single-sided** circuit board, the circuit output as potential difference between two critical points was measured as:

Y = Voltage as millivolt (mv)

The target voltage is 250 mv with +/- 20 mv. When the circuit is rejected due to performance outside the limits, the average rework costs is $42. The optimum nominal values of the factors determined from parameter design studies and the existing tolerances are as shown below.

Figure T.1 Factors, Nominal Values and Tolerances

Notation	Factor Description	Nominal Value	Tolerance (σ)
A	Resistor – Left	40K Ohms	5%
B	Resistor - Right	120K Ohms	5%
C	Transistor	160 (hFE)	50
D	Oscillator	555	5%
E	Jumper Pin	0.80	5%



The quality/grade of the parts under study can be upgraded to better quality (lower tolerances) with costs shown in the table below.

Figure T.2 Upgrade Costs and Tolerances

Type of Part	Grades	Cost	Tolerance (σ)
Resistor	Low grade High Grade	Base Price 1.75	5% 1%
Transistor	Low grade High Grade	Base Price 3.00	50 25
Oscillator	Low grade High Grade	Base Price 1.90	5% 2%
Jumper Pin	Low grade High Grade	Base Price 1.20	5% 1%

TOLERANCE DESIGN

Step 1: Run experiments setting factors at current tolerance limits and perform ANOVA.

To study the 5 factors, an experiment is designed using an L-8 array. The five factors A, B, C, D, and E each are assigned two levels of the factor as shown below:

Level 1 = Nominal value – Tolerance **(σ)**
Level 2 = Nominal value + Tolerance **(σ)**

Note: When a factor is assigned 3 levels, the nominal value makes one level and the other two levels are assigned Nominal +/- $\sqrt{3/2}$ x Tolerance.

Figure T.3 Factor Levels Studied

	Factors	Level 1	Level 2
1	A: Resistor - L	36K Ohms	44K Ohms
2	B: Resistor - R	114K Ohms	126K Ohms
3	COLUMN UNUSED	*UNUSED*	------------
4	C: Transistor	152 hFE	168 hFE
5	D: Oscillator	527 Rated	577 Rated
6	COLUMN UNUSED	*UNUSED*	------------
7	E: Jumper Pin	0.76 mm D	0.84 mm Dia.

Figure T.4 Orthogonal Array and Experimental Results

	A	B		C	D		E	
	1	2	3	4	5	6	7	Results
1	1	1	0	1	1	0	1	246
2	1	1	0	2	2	0	2	259
3	1	2	0	1	1	0	2	232
4	1	2	0	2	2	0	1	243
5	2	1	0	1	2	0	2	279
6	2	1	0	2	1	0	1	260
7	2	2	0	1	2	0	1	275
8	2	2	0	2	1	0	2	240

Figure T.6 Average Effects of Factor Levels (Qualitek-4 software)

Column # / Factors	Level 1	Level 2
1 A: Resistor - L	245	263.5
2 B: Resistor - R	261	247.5
4 C: Transistor	258	250.5
5 D: Oscillator	244.5	264
7 E: Jumper Pin	256	252.5

ANOVA Calculations

Correction Factor

$$CF = (246 + 259 + 232 + 243 + 279 + 260 + 275 + 240)^2/8$$
$$= 517{,}144$$

$$S_T = 246^2 + 259^2 + 232^2 + 243^2 + 279^2 + 260^2 + 275^2 + 240^2 - CF$$
$$= 519{,}136 - 517{,}144$$
$$= 1992$$

(Note that the hand calculations show difference in round-off from Qualitek-4 software out put shown in Fig. T.7)

Since the Degrees of Freedom is 7, from Sums of Squares, $\mathbf{S_T}$, Variance can be calculated as:

$$V_T = 1992/7 = 284.6$$

Individual factor sums of squares can now be calculated for the significant factors as: (Factor average effects needed for factor sums of squares are shown in Fig. T.6)

$$S_A = 4 \times (245^2 + 263.5^2) - CF$$ (4 results are used to calculate average effects)
$$= 517{,}829 - 517{,}144$$
$$= 685$$

$$S_B = 4 \times (261^2 + 247.5^2) - CF$$
$$= 517{,}509 - 517{,}144$$
$$= 365$$

$$S_C = 4 \times (258^2 + 250.5^2) - CF$$
$$= 517{,}257 - 517{,}144$$
$$= 113$$

$$S_D = 4 \times (244^2 + 264^2) - CF$$
$$= 517{,}905 - 517{,}144$$
$$= 761$$

$$S_E = 4 \times (256^2 + 252.5^2) - CF$$
$$= 517{,}169 - 517{,}144$$
$$= 25$$ (This factor is pooled or considered insignificant)

The error term is formed by pooling factor E and the effects from the two empty columns.

$$\mathbf{S_e} = \mathbf{S_T} - (\text{ Sum of squares of all significant factors, A, B, C \& D})$$
$$= 1992 - (685+365+113 + 761)$$
$$= 1992 - 1924$$
$$= 68$$

The pooled degrees of freedom (DOF), f_e becomes:

$$f_e = \text{Total DOF} - (\text{DOF of all significant factors, A, B, C \& D})$$
$$= 7 - 4$$
$$= 3$$

Which gives the error variance, V_e, as**:**

$$\mathbf{V_e} = \mathbf{S_e} / f_e \ \mathbf{= 68 /3 = 23} \ \text{(approximately)}$$

From the error sums of squares and DOF, the pure sum of squares, S', for the factors can be calculated.

$$\mathbf{S_A'} = \mathbf{S_A} - \mathbf{V_e} \times f_A$$
$$\mathbf{= 685 - 23 \times 1}$$
$$\mathbf{= 662}$$

which allows calculation of the relative contribution of factor A as:

$$\mathbf{P_A} = \mathbf{S_A'} / \mathbf{S_T}$$
$$\mathbf{= 662/1992}$$
$$\mathbf{= 33\%}$$

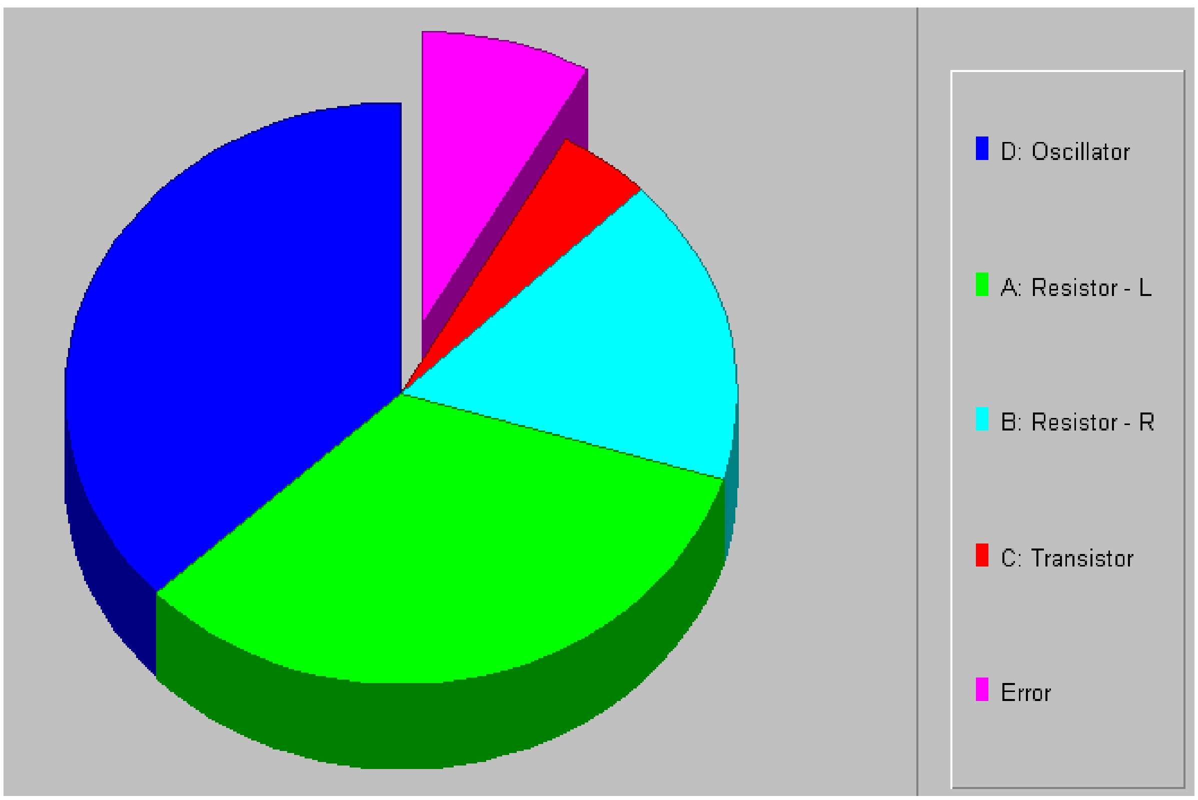
Similarly, relative percent influence of all other factors is calculated as shown in the software output of ANOVA in Fig. T.7 and Fig. T.7a.

(P_A = 33.2%, P_B = 17.1%, P_C = 4.4%, and P_D = 37.0%)

Figure T.7 ANOVA

Col # / Factor	DOF (f)	Sum of Sqrs. (S)	Variance (V)	F - Ratio (F)	Pure Sum (S')	Percent P(%)
1 A: Resistor - L	1	684.5	684.5	29.546	661.333	33.207
2 B: Resistor - R	1	364.5	364.5	15.733	341.333	17.139
4 C: Transistor	1	112.5	112.5	4.856	89.333	4.485
5 D: Oscillator	1	760.5	760.5	32.827	737.333	37.024
7 E: Jumper Pin	(1)	(24.5)		POOLED	(CL=63.46%)	
Other/Error	3	69.5	23.166			8.145
Total:	7	1991.5				100.00%

Figure T.7a Pie Chart Showing Relative Influence of Significant Factors



Step 2: Calculate loss for the current system performance

The system response/output in this case, y = potential difference (mv), the tolerance, TOL = 20, and the average repair cost is $42 (L_c). This specified values allows calculation of the loss constant, K, as:

$$\begin{aligned} \mathbf{K} &= \mathbf{(L_c)/(TOL)2} \\ &= \mathbf{42/((20)^2} \\ &= \mathbf{0.105} \end{aligned}$$

[L = K (MSD), or L = K V_T , where V_T = ST /(DOF)$_T$]

Using the loss function in terms of variance, the current loss (from all factors/components) for the system now can be written as:

$$\begin{aligned} \mathbf{L}_{\text{T(Current)}} &= \mathbf{0.105 \times V_T} \\ &= \mathbf{0.105 \times 284.6} \\ &= \mathbf{29.9} \text{ (\$/Circuit board)} \end{aligned}$$

Step 3: Determine potential savings (loss after – loss before) by adjusting factor levels:

- *Calculate current loss using ANOVA (P%)*
- *Estimate the new loss with upgraded factor level (better grade of materials, part, etc.)*

3a. Current loss from Factors Using ANOVA (P%)

Since loss is proportional to the variance and since the relative percentage of factor influences (in right column in ANOVA) is based on the individual factor variances, the contribution to the total loss by individual factor can be calculated using the percentage of influence.

$$\begin{aligned} \mathbf{L_A} &= \mathbf{L}_{\text{T(Current)}} \mathbf{\times P_A} \\ &= \mathbf{29.9 \times 33.2\%} \\ &= \mathbf{29.9 \times 0.332} \\ &= \mathbf{9.93} \text{ (\$/Circuit board)} \end{aligned}$$

$$
\begin{aligned}
\mathbf{L_B} &= \mathbf{L}_{\text{T(Current)}} \mathbf{\ x\ P}_{\text{B}} \\
&= \mathbf{29.9 \text{ x } 17.1\%} \\
&= \mathbf{29.9 \text{ x } 0.171} \\
&= \mathbf{5.11} \text{ (\$/Circuit board)}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{L_C} &= \mathbf{L}_{\text{T(Current)}} \mathbf{\ x\ P}_{\text{C}} \\
&= \mathbf{29.9 \text{ x } 4.4\%} \\
&= \mathbf{29.9 \text{ x } 0.044} \\
&= \mathbf{1.32} \text{ (\$/Circuit board)}
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{L_D} &= \mathbf{L}_{\text{T(Current)}} \mathbf{\ x\ P}_{\text{D}} \\
&= \mathbf{29.9 \text{ x } 37\%} \\
&= \mathbf{29.9 \text{ x } 0.37} \\
&= \mathbf{11.06} \text{ (\$/Circuit board)}
\end{aligned}
$$

3b. New Loss with Upgraded Parts

The cost of upgraded parts and the new tolerances for each significant factor are shown in Fig. T.8 below. This data can be used to calculate the potential savings when parts of better grade are incorporated in the design. For example, when part is replaced by one with better grade at a cost of $1.75, the savings from such design can be calculated as:

$$
\begin{aligned}
\mathbf{L_{A(New)}} &= \mathbf{L}_{\text{A(Current)}} \mathbf{\ x\ (1\% \ / \ 5\% \ )^2} \quad \text{(Loss is proportional to variance)} \\
&= \mathbf{9.93 \text{ x } 0.04} \\
&= \mathbf{0.40} \text{ (\$/Circuit board, loss is reduce from \$9.93/board)}
\end{aligned}
$$

Thus the quality improvement (QA) from part A in terms of dollars is:

$$
\begin{aligned}
\mathbf{(QA)_A} &= \mathbf{L}_{\text{A(Current)}} \ \mathbf{-} \ \mathbf{L_{A(New)}} \\
&= \mathbf{9.93 \ - 0.40} \\
&= \mathbf{\$ \ 9.53}
\end{aligned}
$$

Of course, this improvement comes at cost of the upgraded part which for part A is $ 1.75. Thus, the net gain from selection of better grade part A is:

$$
\begin{aligned}
\mathbf{(Net\ Gain)_A} &= \mathbf{\$ \ 9.53 \ - \$1.75} \\
&= \mathbf{\$7.78}
\end{aligned}
$$

Similarly, the net gain from all other parts can be calculated as shown below.

$$
\begin{aligned}
\textbf{(Net Gain)}_B &= [L_{B(Current)} - L_{B(Current)} \times (1\% / 5\%)^2] - \text{Upgrade Cost} \\
&= [5.11 - 5.11 \times 0.04] - 1.75 \\
&= [5.11 - 0.20] - 1.75 \\
&= \$3.16
\end{aligned}
$$

$$
\begin{aligned}
\textbf{(Net Gain)}_C &= [L_{C(Current)} - L_{C(Current)} \times (25/50)^2] - \text{Upgrade Cost} \\
&= [1.32 - 1.32 \times 0.25] - 3.00 \\
&= [1.32 - 0.33] - 3.00 \\
&= -\$2.01 \quad \text{(negative savings)}
\end{aligned}
$$

Factor C has a negative savings which does not make it worthwhile to upgrade and will not be considered in the new design.

$$
\begin{aligned}
\textbf{(Net Gain)}_D &= [L_{D(Current)} - L_{D(Current)} \times (2\% / 5\%)^2] - \text{Upgrade Cost} \\
&= [11.06 - 11.06 \times 0.16] - 1.90 \\
&= [11.06 - 1.77] - 1.90 \\
&= \$7.39
\end{aligned}
$$

Figure T.8 Upgrade Costs and Tolerances for Significant Factors

Type of Part	Grades	Cost	Tolerance (σ)
Resistor (A & B)	Low grade High Grade	Base Price 1.75	5% 1%
Transistor (C)	Low grade High Grade	Base Price 3.00	50 25
Oscillator (D)	Low grade High Grade	Base Price 1.90	5% 2%

Step 4: Calculate net gain (4) from savings from all factors at the improved condition.

Based on the potential savings, only factors A, B, and D

$$
\begin{aligned}
\textbf{Total Quality Improvement} &= 9.53 + 4.90 + 9.29 \\
&= 23.72 \ (\$/\text{Circuit board})
\end{aligned}
$$

Total Upgrade Cost (A, B, & D)**= 1.75 + 1.75 + 1.90**

= **5.40** ($/Circuit board)

Total Net Gain = Total Improvement – Cost of Improvement
= 23.72 - 5.40
= 18.32 ($/Circuit board)

Figure T.9 Calculated Loss, Upgrade Cost and Net Gain Data

	Factor	P%	Loss - current	Loss – New $	Saving $	Upgrade Cost $	Net Gain $	Comment
A	Resistor – L	33.2	9.93	0.40	9.53	1.75	7.78	Upgraded
B	Resistor – R	17.1	5.11	0.20	4.90	1.75	3.16	Upgraded
C	Transistor	4.4	1.32	0.33	0.99	3.00	-2.01	Not upgraded
D	Oscillator	37	11.06	1.77	9.29	1.90	7.39	Upgraded
E	Jumper Pin	0	0	--	--	1.20	--	
	TOTAL	**100**	**29.9**		**23.72**	**5.40**	**18.32**	

If the monthly production of this business unit is 4,500 Circuit boards per month (production volume), There will be potential to save 4,500 x $18.32 = $82,440 per month from this improvement obtained from TOLERANCE DESIGN study.

Considerations for Applying Tolerance Designs for Your System

1. Completed optimizing your product or process using *parameter design*.
2. Identify a key response characteristic for which you know the TOLERANCE and the LOSS sustained when performance is out of specification limits.
3. Identify factors to study for which there are better quality (grades) alternatives available at a cost.
4. Have resources to carry out simpler experiments designed using orthogonal arrays
5. Know how to perform ANOVA on results and be familiar with *loss function*.